NOMOGRAPH CONSTRUCTION METHODS

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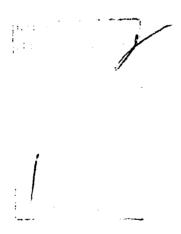
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NOMOGRAPH CONSTRUCTION METHODS

ROBERT R. YEAGER
Technology Incorporated

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FOREWORD

Technology Incorporated, Dayton, Ohio, prepared this report on nomograph construction methods as an integral part of a contractual effort intended primarily to devise a stress analysis manual. All work was authorized under Contract F33615-67-C-1538, which was initiated and sponsored by the Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio, under Project 1467 "Structural Analysis Methods", Task No. 146702 "Thermoelastic Structural Analysis Methods". The contractual period extended from 30 April 1967 to 30 April 1969.

Mr. Gene E. Maddux, of the Air Force Flight Dynamics Laboratory, served as the Air Force contract monitor. For Technology Incorporated,

Mr. Dudley C. Ward, manager of the Aeromechanics Department, was the project director; and Mr. Leon A. Vorst, senior research engineer, was the project engineer.

The author is grateful for the direction and assistance of Mr. Maddux; and of Messrs. Thomas J. Hogan and Harold P. Zimmerman, both scientific programmers of Technology Incorporated.

This technical report has been reviewed and is approved.

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Structures Division

Air Force Flight Dynamics Laboratory

ABSTRACT

This report presents nomographic methods developed to graphically represent the various functional relationships between three or more parameters interrelated by equations. Also included are methods developed to vary both the form and the size of given nomographs. These developments were based on colinear techniques.

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1. INTRODUCTION

The fundamental principles underlying nomographic construction have been understood for many years. However, the development and application of nomography has been limited in the past because of misunderstandings in regard to the severity of the construction procedures.

This report furnishes the basic information necessary to the construction of nomographs. In addition, it explains the role of normal and radial graphing techniques as supplementary aids to nomography in describing mathematical functions.

2. NOMOGRAPHY AND ITS EMPLOYMENT

A nomograph is a two-dimensional, pictorial representation of a mathematical relationship. This relationship incorporates three or more variables which are functionally describable and void of excluded values within their numerical ranges. A nomograph, utilizing colinear concepts, connects curves depicting these variables with straight lines which enable a value of the dependent variable to be determined for selected values of the independent variables.

It is important to realize that a nomograph has limited applicability.

Any equation which is not commonly used by engineering or design personnel will not justify the time and effort necessary to the construction of its nomograph. Equations which are simply constructed with a minimum number of variables can be easily handled within an individual's mind whereas extremely complex equations requiring major nomographic efforts can usually be simulated by computer programs with less difficulty.

Nomographs should be constructed for the repetitious equations used by the specialized engineer as well as for the generalized equations found in many areas of interest. The construction of slide rules and radial graphs are but two of the many complementary benefits derived from an understanding of nomographic principles.

3. BASIC PRINCIPLES IN NOMOGRAPHIC CONSTRUCTION

A basic understanding of the fundamental principles underlying nomography may be achieved by first considering the graph in Figure 1(a).

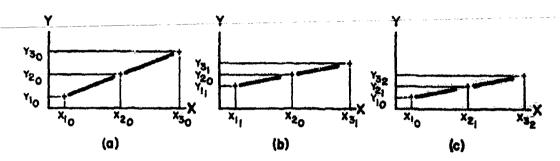


Figure 1. Basic Colinear Relationships in Nomograms

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It is apparent that the three points on the straight line have a colinear relationship which can be expressed in the determinate form of

$$\begin{bmatrix} X_{10} & Y_{10} & I \\ X_{20} & Y_{20} & I \\ X_{30} & Y_{30} & I \end{bmatrix} = 0$$
 (1)

where each element of the determinant has a specific numerical value. Determinants for Figures 1(b) and (c) can similarly be constructed.

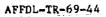
$$X_{i_1}$$
 Y_{i_1} I = 0 (2)
 X_{2_0} Y_{3_1} I

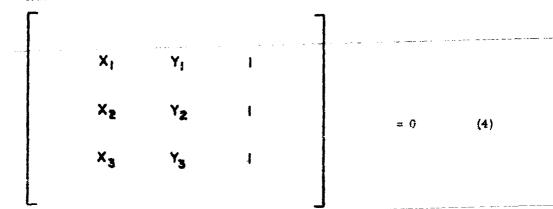
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$$\begin{bmatrix} X_{10} & Y_{10} & I \\ X_{21} & Y_{21} & I \\ X_{32} & Y_{32} & I \end{bmatrix} = 0$$
 (3)

Analysis of determinants (1) and (2) and determinants (1) and (3) reveals the interdependency of the positions of the three points

Consider now the determinant





where

$$X_1 = f_1(A) \qquad Y_1 = f_2(A)$$

$$X_2 = g_1(B)$$
 $Y_2 = g_2(B)$

$$X_3 = h_1(C)$$
 $Y_3 = h_2(C)$

The solution of determinant (4) in terms of arbitrarily selected functions will yield an equation in terms of A, B, and C. The selection of the numerical range of values for any two of these variables will automatically dictate the range of values for the third variable if the zero equivalency of the determinant is to be maintained.

Thus, determinant (4) is the generalized version of determinants (1), (2) and (3) and represents an infinite number of colinear relationships between defined points in the XY plane.

The graphical relationship of the X and Y functions to the nomograph can now be established. To simplify this procedure, the independent variable (A)

in functions f_1 and f_2 will be replaced by an arbitrarily defined variable (Z) which has the same numerical boundary range as (A). The coordinates of X_1 and Y_1 can be calculated by substituting the values of Z into the functions f_1 and f_2 . These coordinates are then plotted on an X, Y, Z graph similar to that shown in Figure 2.

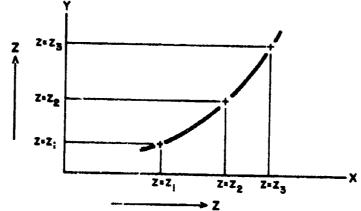


Figure 2. Graph Transform of X, Y Coordinates for a Single X, Y Function

This procedure may be repeated for the elements in the second and third rows of determinant (4), and a set of curves may be established on the X, Y, Z coordinate graph depicted in Figure 3.

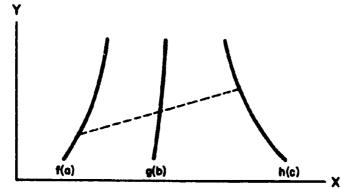


Figure 3. Graph Transform of X, Y Coordinates for Three X, Y Functions

These three curves represent a structural relationship and this pictorial kinship is called a nomograph. Note that the points defined by the intersection of any straight line with these curves will satisfy the numerical requirements of determinant (4).

The following example will serve to illustrate the discussed procedure.

Knowing that

$$\begin{bmatrix} x_1 & Y_1 & 1 \\ x_2 & Y_2 & 1 \\ x_3 & Y_3 & 1 \end{bmatrix} = 0$$

and arbitrarily defining

$$X_1 = f_1(A) = -1.0$$
 $Y_2 = f_4(B) = -B/4.0$
 $Y_1 = f_2(A) = -A$ $X_3 = f_5(C) = 1.0$
 $X_2 = f_3(B) = 0.0$ $Y_3 = f_6(C) = C/4.0$

the following determinant can be constructed.

Solving for A

$$A = B/2.0 + C/4.0$$

Normally the range of the numerical values for B and C would be determined from physical criteria. In this example, however, they will be defined as

$$0.0 \le B \le 4.0$$

$$0.0 \le C \le 4.0$$

Thus,

$$0.0 \le A \le 3.0$$

The nomograph for this equation can now be plotted. The notation of replacing the variables with Z will be dropped since it only established the process of presenting all curves on the same graph.

It is important to remember that the determined numerical values for the coordinates of the curves are not the numerical values of the variable. This is clearly demonstrated in Figure 4.

$X_1 = -1.0 Y_1 = -A$ $0.0 \le A \le 3.0$ $A X_1 Y_1$ $0.0 -1.0 0.0$		$X_2 = 0.0 Y_2 = -B/4.0$ $0 \le B \le 4.0$			$X_3 = 1$ $Y_3 = C/4.0$ $0 \le C \le 4.0$			
A	\mathbf{x}_1	Yı	В	X2	Y _Z	0	x_3	Y3
0.0	-1.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0
1.0	-1.0	-1.0	1.0	0.0	25	1.0	1.0	. 25
2.0	-1.0	.2.0	2.0	0.0	50	2.0	1.0	. 50
3.0	-1.0	-3.0	3.0	0.0	75	3.0	1.0	. 75
			4.0	0.0	-1.00	4.0	1.0	1.00

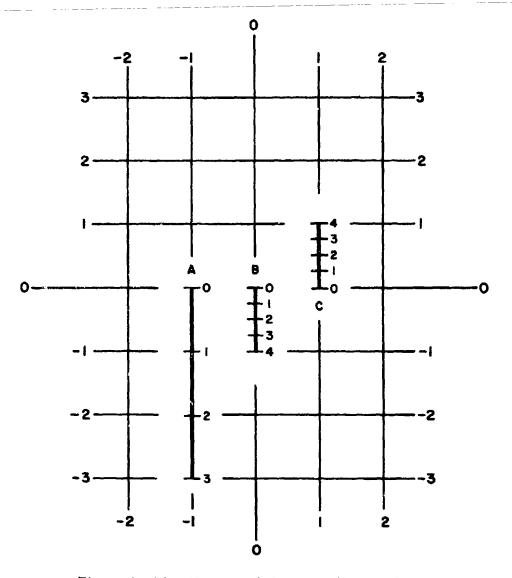


Figure 4. Plan Nomograph for A = B/2.0 + C/4.0

It is now possible to use the nomograph as shown in Figure 4 with the equation

$$A \approx B/2.0 + C/4.0$$

by selecting values of B and C within the ranges indicated for such by the nomograph. For example, choosing C = 4 and B = 2 and drawing a line through these points permits A to be determined as equaling 2. (See Figure 4(a))

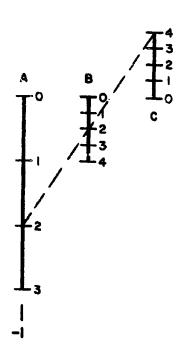


Figure 4(a) Working Nomograph for A = B/2.0 + C/4.0

3.1 Sizing Techniques

Any nomograph can usually be modified in regard to its shape and relative position of curves. This enables information to be presented in a format which is more acceptable. There are a number of processes by which this can be accomplished. One such method utilizes the digital computer and specific mathematic transforms which attempt to modify any determinant in the construction form of determinant (4) (See reference 1). A second method consists of determining coefficients which nondimensionalize the elements within the determinant. This method is illustrated in the following example.

Defining the determinant,

$$\begin{bmatrix} -A & A^2 & I \\ 0 & B & I \\ C & C^2 & I \end{bmatrix} = 0$$
 (6)

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and solving for B, yields the fundamental mathematic expression

B = AC

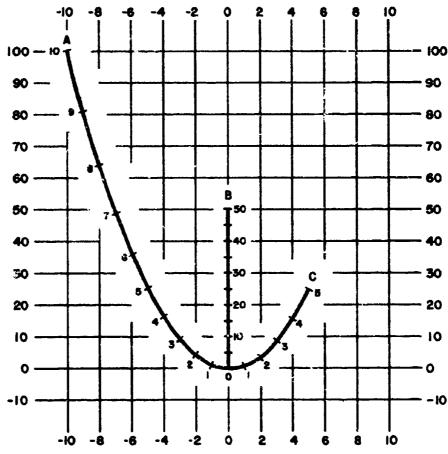
Boundary ranges for A and C are arbitrarily selected.

$$0 \le A \le 10$$

Thus,

 $0 \le B \le 50$

Figure 5 illustrates the resulting nomograph.



However, the equation

can be modified to

$$\frac{B}{50} = \frac{A}{10} \times \frac{C}{5}$$

using the selected variable ranges.

The elements in determinant (6) can now be replaced by the modified forms of the variables.

$$\begin{bmatrix} -\frac{A}{10} & (\frac{A}{10})^2 & 1 \\ 0 & \frac{B}{50} & 1 \\ \frac{C}{5} & (\frac{C}{5})^2 & 1 \end{bmatrix} = 0 \tag{7}$$

Figure 6 depicts the resulting modified determinant.

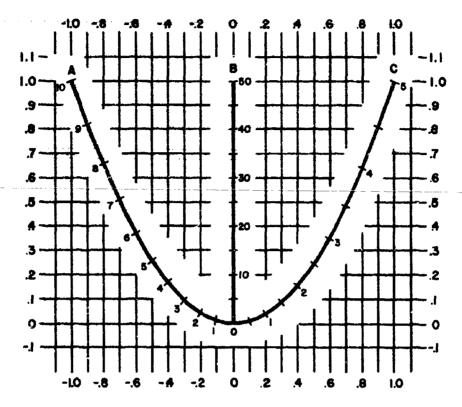


Figure 5. Modified Plan Nomograph for B = AC

The shape of this nomograph permits a more general usage and its scales are easier to read.

A third method by which the shape of nomographs may be changed is established by altering the size of the units used in the pictorial construction of the nomograph. Figure 7 illustrates this mechanism for the equation used in determinant (7).

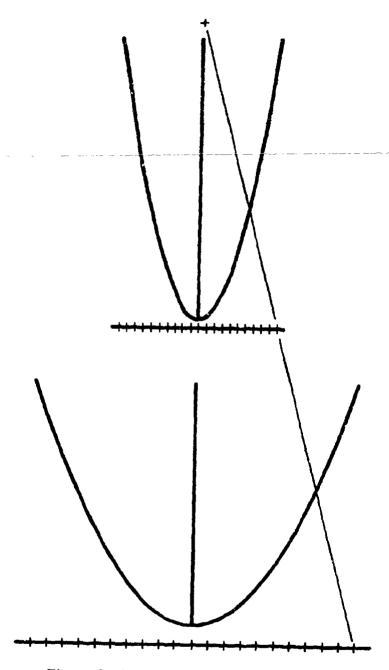


Figure 7. Nomographic Sizing Technique

Skill in shaping nomographs will be acquired by interested personnel as their experience enlarges. The ability to transform the basic determinant into one which is more usable depends upon the individual rather than a mathematical or mechanical procedure.

3.2 Composite Nomographs

There are two possible ways in which to pictorially present the mathematical relationship between more than three variables. One procedure is to treat the combination of two variables as one. For example,

$$A \approx B + CD$$

can be represented by

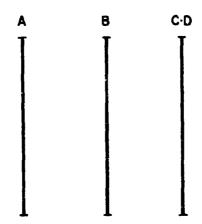


Figure 8. Nomograph of A = B + CD

A second procedure, which enables each variable to be individually portrayed, incorporates the fabrication of dummy parameters. Consider

A = BC + DE

Defining

$$N_1 = BC$$

$$N_2 = DE$$

Thus, establishing $A = N_1 + N_2$. The following determinants can be constructed.

The nomographs for these determinants can now be combined as shown in Figure 9.

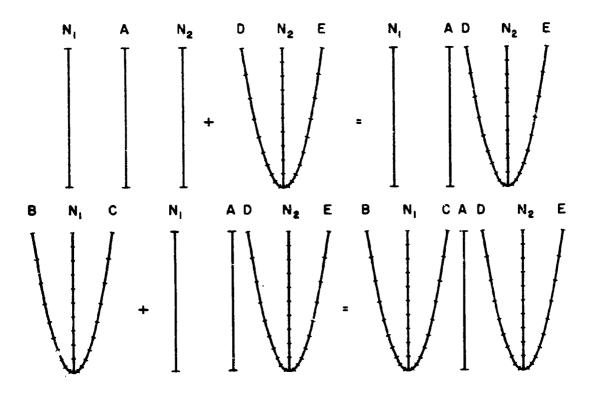
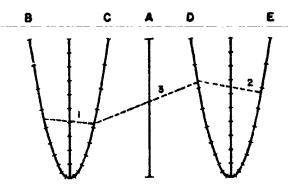


Figure 9. Integration of Several Nomographs

Analysis of the final nomograph will show that N_1 and N_2 are dummy lines and need no scales. In addition, it is important to note that the order of linking is now more complex and should be indicated in an auxiliary diagram.



3.3 Normal and Radial Graphing Aids

Normal two axis graphs can be attached to nomographs. This usually requires a linearly scaled nomographic line which is identical to one of the axes of the normal graph. An example of this is illustrated in Figure 10.

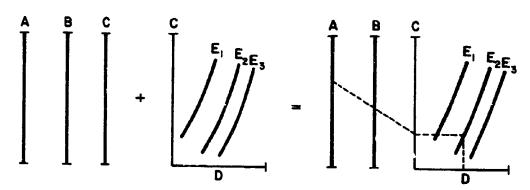


Figure 10. Integration of Normal Graphs and Nomographs
Scales on line C are not required.

Interesting two variable radial graphs can be constructed from determinate form by replacing both the X and Y elements of any row with specific numerical values. This results in a two variable relationship which is very useful in several ways. First, it permits normal graphs to be replaced by a graph which is more easily used. For example,

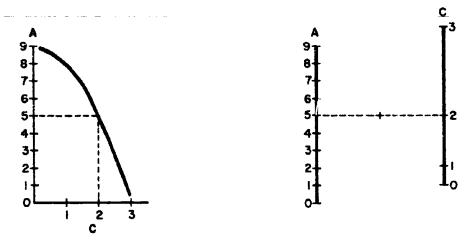


Figure 11. Transformation of Normal Graphs into Nomographs
Secondly, it permits two nomographs with identical variables to be linked.

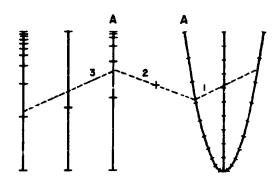


Figure 12. Nomographic Linkage

To maintain the required colinear relation, a focal point must exist which is valid for all lines passing through the two curves. The coordinates for this point are determined by the values of the numerical elements in the replaced row.

3.4 Nomographic Determinants

The following list of determinants and equations is presented in order to provide a supplementary digest of existing nomographs.

$$A = \frac{B^2 + C^2}{B + C}$$

0
$$\frac{2}{3A}$$
 | $\frac{1}{B^2}$ | $\frac{2}{3(B+1)}$ | $\frac{2}{3C}$ |

$$B^2 + AB + C = 0$$

O A
$$\frac{B^2}{B^2+1} = \frac{-B}{B^2+1}$$

$$CB + A/B + 1 = 0$$

$$B^2 = \frac{A+C}{A-C}$$

$$A + C = \frac{1}{B}$$

$$B = \frac{A + C}{AC}$$

$$\frac{1}{A} \qquad A^2 \qquad 1$$

$$0 \qquad B^2 \qquad 1$$

$$\frac{1}{C} \qquad C^2 \qquad 1$$

$$B^2 = C^2 + AC + A^2$$

$$-\frac{1}{A^2} \qquad \frac{1}{A}$$

$$C \qquad \frac{1}{B}$$

$$\frac{1}{C^2} \qquad \frac{1}{C}$$

$$B = \frac{A^2 + C^2}{A + C}$$

$$\frac{1}{A} \qquad -\frac{1}{A^2}$$

$$0 \qquad \qquad \frac{1}{B}$$

$$\frac{1}{C} \qquad -\frac{1}{C^2}$$

$$A^{2}$$
 A^{2} 1
 $1 + \frac{B}{2}$ $\frac{B}{2}$ 1
 $C^{2} + 2$ C^{2} 1

$$B = A^2 + C^2$$

$$\frac{1}{B} = \frac{1}{A} + \frac{1}{C}$$

$$C : \frac{A^2 - AB^2}{A^2 - B^2}$$

$$C = \frac{A}{1 - B^3}$$

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- 1. Hogan, Thomas J., Automated Nomography, Technology Incorporated, 1968.
- 2. Allcock, H.J., Jones, J. Reginald, and Michel, J.G.L., The Nomogram, Pitman, New York, 1963.

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